$$\sum_{r=1}^{n} \left(a(p(i_r)) - y_r \right) \cdot a'(p(i_r)) \cdot p'(i_r) = 0;$$

$$a(x) = \frac{x+|x|}{2}; \quad a'(x) = \frac{1+\mathrm{sign}(x)}{2}$$

$$\sum_{r=1}^{n} \left(\frac{p(i_r)+|p(i_r)|}{2} - y_r \right) \cdot \frac{1+\mathrm{sign}(p(i_r))}{2} \cdot p'(i_r) = 0;$$

$$\sum_{r=1}^{n} \left(\frac{p(i_r)+|p(i_r)|-2y_r}{2} \right) \cdot \frac{1+\mathrm{sign}(p(i_r))}{2} \cdot p'(i_r) = 0;$$

$$\sum_{r=1}^{n} \frac{(p(i_r)+|p(i_r)|-2y_r)(1+\mathrm{sign}(p(i_r)))\cdot p'(i_r)}{\cancel{4}} = 0;$$

$$\sum_{r=1}^{n} \left(p(i_r)+|p(i_r)|-2y_r \right)\left(1+\mathrm{sign}(p(i_r))\right)\cdot p'(i_r) = 0;$$

$$\sum_{r=1}^{n} \left(p(i_r)p'(i_r) + |p(i_r)|\,p'(i_r) - p'(i_r)\cdot 2y_r \right) \underline{\left(1+\mathrm{sign}(p(i_r))\right)} = 0;$$

$$\boxed{\mathrm{sign}(p(i_r)) = -1? \Rightarrow \; = 0}$$

$\mathrm{sign}(p(i_r)) = 1? \Rightarrow |p(i_r)| = p(i_r)$, then

$$\sum_{r=1}^{n} \left(p(i_r)p'(i_r) + p(i_r)p'(i_r) - p'(i_r)\cdot 2y_r \right)\left(1+\mathrm{sign}(p(i_r))\right) = 0;$$

$$\sum_{r=1}^{n} \left(2p(i_r)p'(i_r) - 2p'(i_r)\cdot y_r \right)\left(1+\mathrm{sign}(p(i_r))\right) = 0;$$

$$\sum_{r=1}^{n} \left(p(i_r)p'(i_r) - p'(i_r)\,y_r \right)\left(1+\mathrm{sign}(p(i_r))\right) = 0;$$

If $p(i_r) = \sum_{g=1}^{n_w} i_{rg} w_g + b$, then:

for w_k, $p'(i_r) = i_{rk}$,

for b, $p'(i_r) = 1$

MSE with ReLU for w_k:

$$\sum_{r=1}^{n}(p(i_r)\cdot i_{rk}-i_{rk}\cdot y_r)(1+sign(p(i_r)))=0;$$

$$\sum_{r=1}^{n}\Big(\sum_{g=1}^{n_w} w_g i_{rg} i_{rk}+b\cdot i_{rk}-i_{rk}\cdot y_r\Big)(1+sign(p(i_r)))=0;$$

$$\sum_{r=1}^{n}\Big(w_k (i_{rk})^2+\sum_{g=1, g\neq k}^{n_w} w_g i_{rg} i_{rk}+b\cdot i_{rk}-y_r i_{rk}\Big)(1+sign(p(i_r)))=0;$$

$$\sum_{r=1}^{n}\Big(w_k (i_{rk})^2-\Big(y_r i_{rk}-\sum_{g=1, g\neq k}^{n_w} w_g i_{rg} i_{rk}-b\, i_{rk}\Big)\times$$

$$\times(1+sign(p(i_r)))=0;$$

$$\sum_{r=1}^{n}(w_k\cdot i_{rk}^2)(1+sign(p(i_r)))=\sum_{r=1}^{n}\Big(y_r i_{rk}-\sum_{g=1, g\neq k}^{n_w} w_g i_{rg} i_{rk}-b\cdot i_r k\Big)\times$$

$$\times(1+sign(p(i_r)));$$

$$\sum_{r=1}^{n}(w_k\cdot i_{rk}^2)(1+sign(p(i_r)))=\sum_{r=1}^{n} i_{rk}\Big(y_r-\sum_{g=1, g\neq k}^{n_w} w_g i_{rg}-b\Big)\times$$

$$\times(1+sign(p(i_r)));$$

Let $1+sign(p(i_r))=z(i_r)$, for simplicity;

$z(x)=1+sign(p(x))$

$$\sum_{r=1}^{n} w_k i_{rk}^2 z(i_r)=\sum_{r=1}^{n} i_{rk}\Big(y_r-\sum_{g=1, g\neq k}^{n_w} w_g i_{rg}-b\Big)\cdot z(i_r);$$

$$w_k\cdot\Big(\sum_{r=1}^{n} i_{rk}^2 z(i_r)\Big)=\sum_{r=1}^{n} i_{rk}\Big(y_r-\sum_{g=1, g\neq k}^{n_w} w_g i_{rg}-b\Big)\cdot z(i_r);$$

$$w_k=\frac{\sum_{r=1}^{n} i_{rk}\Big(y_r-\sum_{g=1, g\neq k}^{n_w} w_g i_{rg}-b\Big)\cdot z(i_r)}{\sum_{r=1}^{n} i_{rk}^2 z(i_r)};$$

MSE with ReLU for b:

$$\sum_{r=1}^{n} (p(i_r) - y_r)(1+\text{sign}(p(i_r))) = 0;$$

$$\sum_{r=1}^{n} (\sum_{g=1}^{n_w} w_g i_{rg} + b - y_r)(1+\text{sign}(p(i_r))) = 0;$$

$$z(x) = 1+\text{sign}(p(x))$$

$$\sum_{r=1}^{n} (\sum_{g=1}^{n_w} w_g i_{rg} + b - y_r) \cdot z(i_r) = 0;$$

$$\sum_{r=1}^{n} (b - (y_r - \sum_{g=1}^{n_w} w_g i_{rg})) \cdot z(i_r) = 0;$$

$$\sum_{r=1}^{n} b \cdot z(i_r) = \sum_{r=1}^{n} (y_r - \sum_{g=1}^{n_w} w_g i_{rg}) \cdot z(i_r);$$

$$b(\sum_{r=1}^{n} z(i_r)) = \sum_{r=1}^{n} (y_r - \sum_{g=1}^{n_w} w_g i_{rg}) \cdot z(i_r);$$

$$b = \frac{\sum_{r=1}^{n} (y_r - \sum_{g=1}^{n_w} w_g i_{rg}) \cdot z(i_r)}{\sum_{r=1}^{n} z(i_r)}$$